Design of Engineering Experiments Part 7 – The 2^{k-p} Fractional Factorial Design

- Text reference, Chapter 8
- **Motivation** for fractional factorials is obvious; as the number of factors becomes large enough to be "interesting", the size of the designs grows very quickly
- Emphasis is on **factor screening**; efficiently identify the factors with large effects
- There may be **many** variables (often because we don't know much about the system)
- Almost always run as **unreplicated** factorials, but often with **center points**

Why do Fractional Factorial Designs Work?

- The **sparsity of effects** principle
  - There may be lots of factors, but few are important
  - System is dominated by main effects, low-order interactions
- The **projection** property
  - Every fractional factorial contains full factorials in fewer factors
- **Sequential** experimentation
  - Can add runs to a fractional factorial to resolve difficulties (or ambiguities) in interpretation

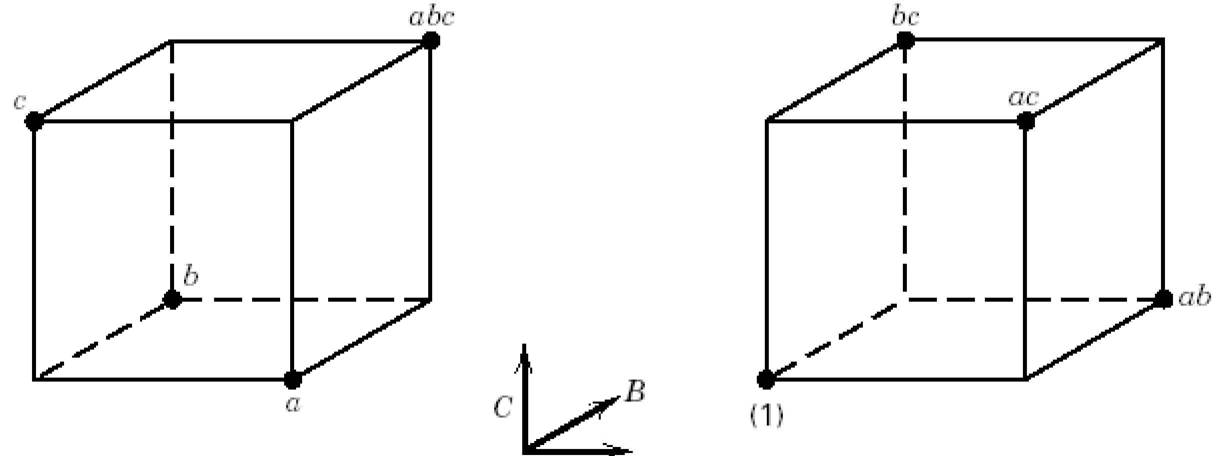
The One-Half Fraction of the 2^k

- Section 8-2, page 283
- Notation: because the design has $2^k/2$ runs, it's referred to as a 2^{k-1}
- Consider a really simple case, the 2^{3-1}
- Note that $I = ABC$

Table 8-1 Plus and Minus Signs for the 2^3 Factorial Design

Treatment Combination	Factorial Effect							
	I	*A*	*B*	*C*	*AB*	*AC*	*BC*	*ABC*
a	+	+	−	−	−	−	+	+
b	+	−	+	−	−	+	−	+
c	+	−	−	+	+	−	−	+
abc	+	+	+	+	+	+	+	+
ab	+	+	+	−	+	−	−	−
ac	+	+	−	+	−	+	−	−
bc	+	−	+	+	−	−	+	−
(1)	+	−	−	−	+	+	+	−

The One-Half Fraction of the 2^3



(*a*) The principal fraction, $I = +ABC$

(*b*) The alternate fraction, $I = -ABC$

Figure 8-1 The two one-half fractions of the 2^3 design.

For the principal fraction, notice that the contrast for estimating the main effect *A* is exactly the same as the contrast used for estimating the *BC* interaction.

This phenomena is called **aliasing** and it occurs in all fractional designs

Aliases can be found directly from the columns in the table of + and - signs

Aliasing in the One-Half Fraction of the 2^3

$A = BC$, $B = AC$, $C = AB$ (or *me* = 2*fi*)

Aliases can be found from the **defining relation** $I = ABC$ by multiplication:

$AI = A(ABC) = A^2BC = BC$

$BI = B(ABC) = AC$

$CI = C(ABC) = AB$

Textbook notation for aliased effects:

$$\ell_A \rightarrow A + BC, \quad \ell_B \rightarrow B + AC, \quad \ell_C \rightarrow C + AB$$

The Alternate Fraction of the 2^{3-1}

- $I = -ABC$ is the defining relation
- Implies slightly different aliases: $A = -BC$, $B = -AC$, and $C = -AB$
- Both designs belong to the same **family,** defined by

$$I = \pm ABC$$

- Suppose that after running the principal fraction, the alternate fraction was also run
- The two groups of runs can be combined to form a full factorial – an example of **sequential** experimentation

Design Resolution

- Resolution III Designs:
  - $me = 2fi$
  - example 2_{III}^{3-1}
- Resolution IV Designs:
  - $2fi = 2fi$
  - example 2_{IV}^{4-1}
- Resolution V Designs:
  - $2fi = 3fi$
  - example 2_{V}^{5-1}

Construction of a One-half Fraction

The **basic** design; the design **generator**

Table 8-2 The Two One-Half Fractions of the 2^3 Design

	Full 2^2 Factorial (Basic Design)		$2_{III}^{3-1}, I = ABC$			$2_{III}^{3-1}, I = -ABC$		
Run	A	B	A	B	$C = AB$	A	B	$C = -AB$
1	−	−	−	−	+	−	−	−
2	+	−	+	−	−	+	−	+
3	−	+	−	+	−	−	+	+
4	+	+	+	+	+	+	+	−

Projection of Fractional Factorials

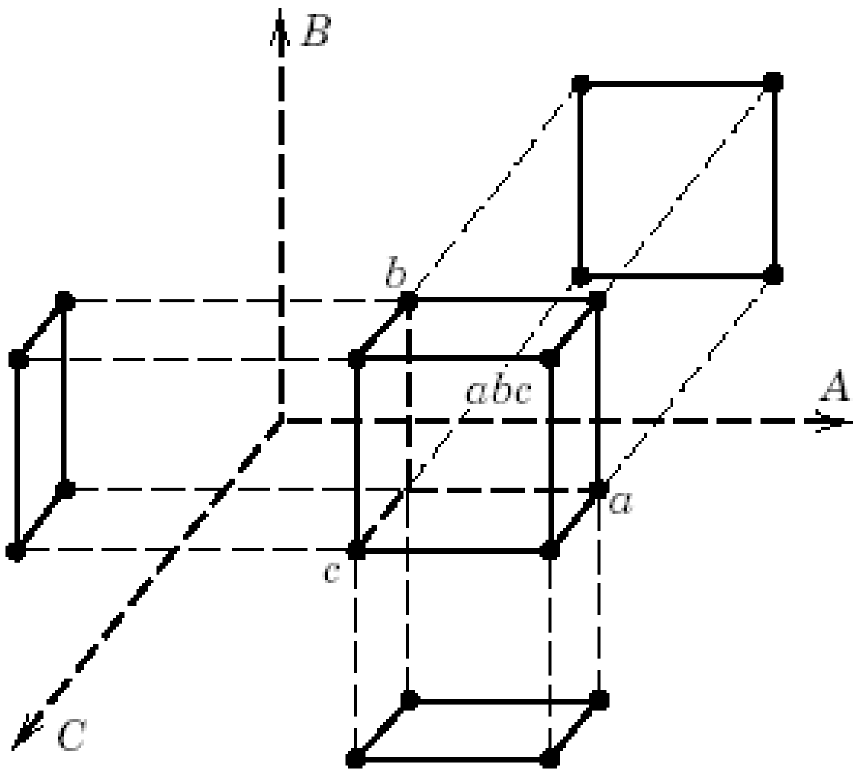


Figure 8-2 Projection of a 2^{3-1}_{III} design into three 2^2 designs.

Every fractional factorial contains full factorials in fewer factors

The "flashlight" analogy

A one-half fraction will project into a full factorial in any $k - 1$ of the original factors

Example 8-1

Table 8-3 The 2_{IV}^{4-1} Design with the Defining Relation $I = ABCD$

	Basic Design					
Run	A	B	C	$D = ABC$	Treatment Combination	Filtration Rate
1	−	−	−	−	(1)	45
2	+	−	−	+	*ad*	100
3	−	+	−	+	*bd*	45
4	+	+	−	−	*ab*	65
5	−	−	+	+	*cd*	75
6	+	−	+	−	*ac*	60
7	−	+	+	−	*bc*	80
8	+	+	+	+	*abcd*	96

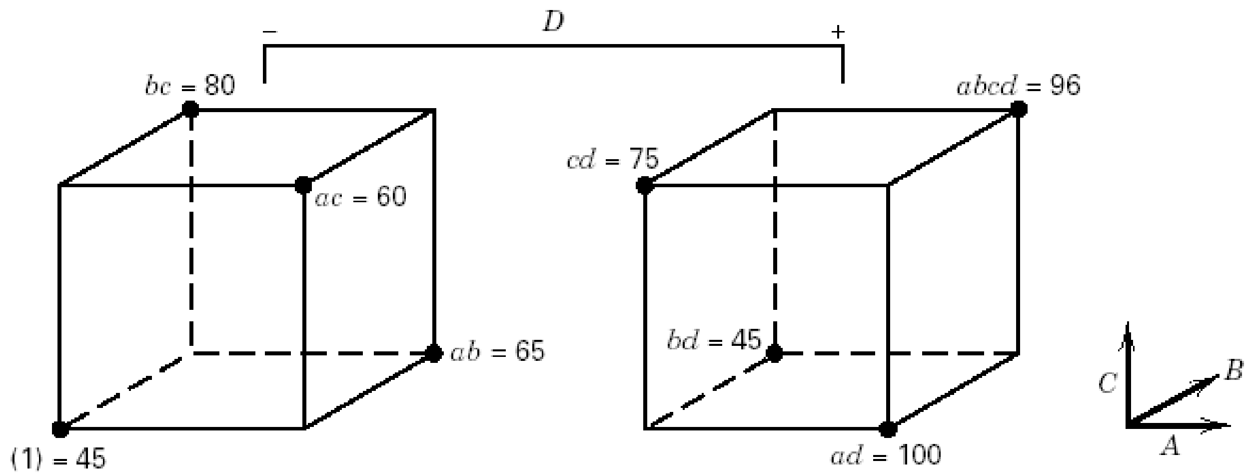


Figure 8-3 The 2_{IV}^{4-1} design for the filtration rate experiment of Example 8-1.

Example 8-1

Interpretation of results often relies on making some assumptions

Ockham's razor

Confirmation experiments can be important

Adding the **alternate fraction** – see page 294

Table 8-4 Estimates of Effects and Aliases from Example 8-1[a]

Estimate	Alias Structure
$[A] = 19.00$	$[A] \rightarrow \mathbf{A} + BCD$
$[B] = 1.50$	$[B] \rightarrow B + ACD$
$[C] = 14.00$	$[C] \rightarrow \mathbf{C} + ABD$
$[D] = 16.50$	$[D] \rightarrow \mathbf{D} + ABC$
$[AB] = -1.00$	$[AB] \rightarrow AB + CD$
$[AC] = -18.50$	$[AC] \rightarrow \mathbf{AC} + BD$
$[AD] = 19.00$	$[AD] \rightarrow \mathbf{AD} + BC$

[a]Significant effects are shown in boldface type.

The *AC* and *AD* interactions can be verified by inspection of the cube plot

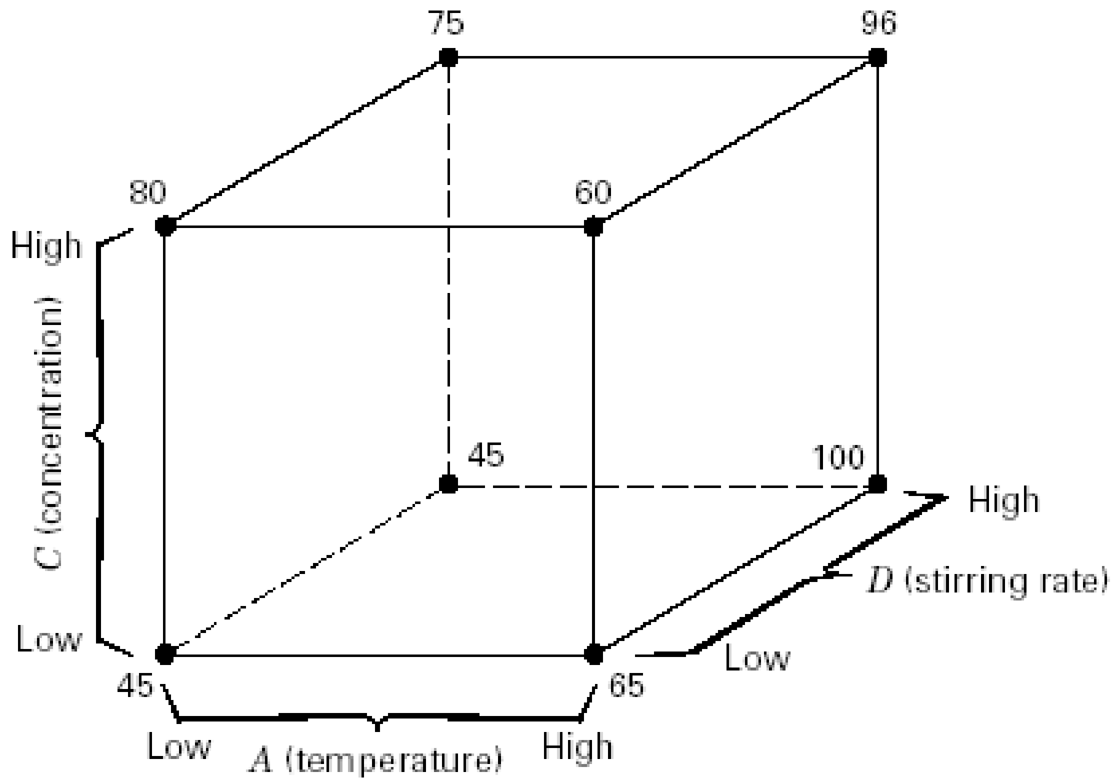


Figure 8-4 Projection of the 2^{4-1}_{IV} design into a 2^3 design in A, C, and D for Example 8-1.

Confirmation experiment for this example: see pages 295-296

Use the model to predict the response at a test combination of interest in the design space – not one of the points in the current design.

Run this test combination – then compare predicted and observed.

For Example 8-1, consider the point +, +, -, +. The predicted response is

$$\hat{y} = 70.75 + \left(\frac{19.00}{2}\right)x_1 + \left(\frac{14.00}{2}\right)x_3 + \left(\frac{16.50}{2}\right)x_4 + \left(\frac{-18.50}{2}\right)x_1x_3 + \left(\frac{19.00}{2}\right)x_1x_4$$

$$= 70.75 + \left(\frac{19.00}{2}\right)(1) + \left(\frac{14.00}{2}\right)(-1) + \left(\frac{16.50}{2}\right)(1) + \left(\frac{-18.50}{2}\right)(1)(-1)$$

$$+ \left(\frac{19.00}{2}\right)(1)(1)$$

$$= 100.25$$

Actual response is 104.

Possible Strategies for Follow-Up Experimentation Following a Fractional Factorial Design

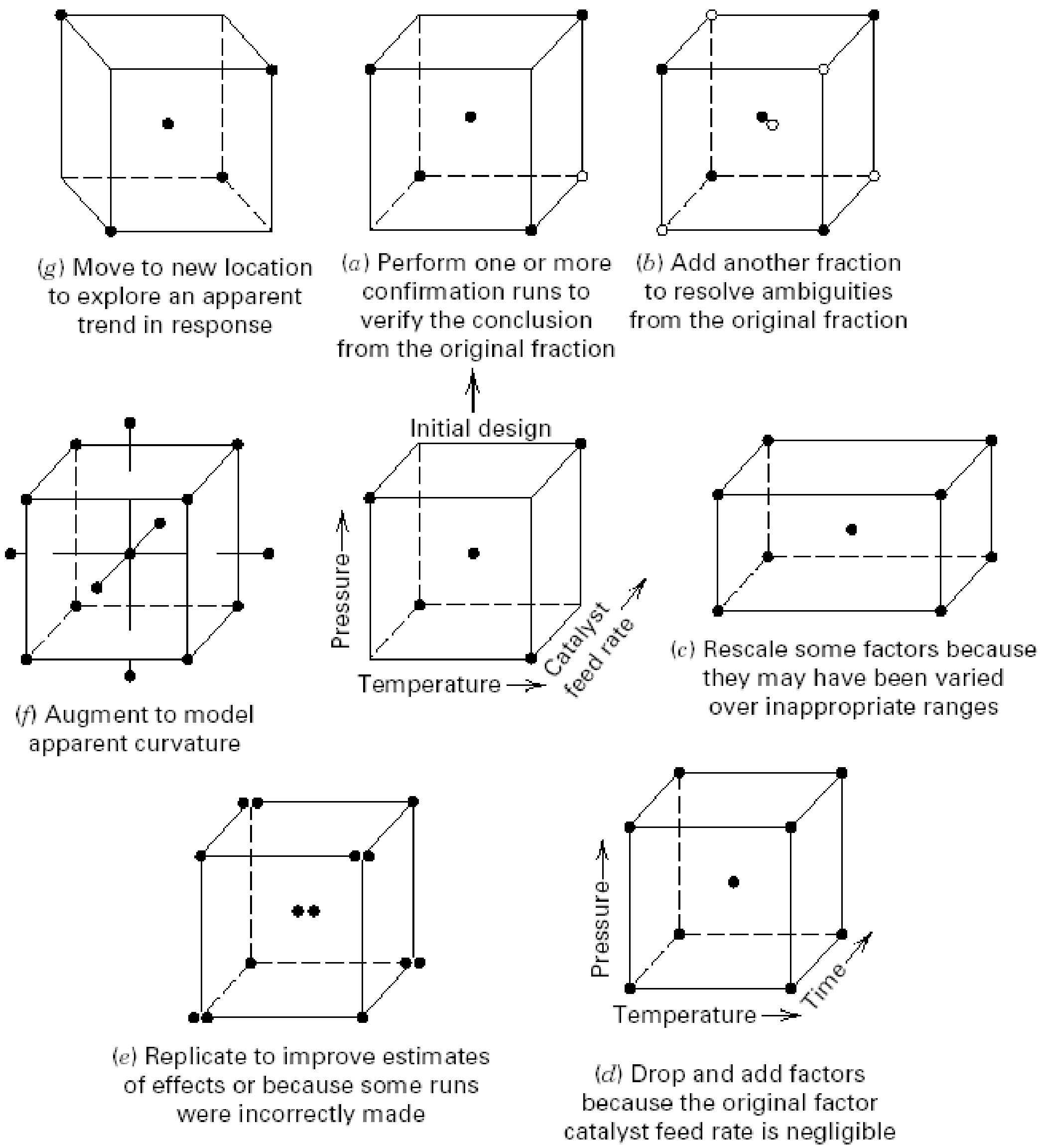


Figure 8-11 Possibilities for follow-up experimentation after a fractional factorial experiment (adapted from Box (1992–93), with permission of the publisher).

The One-Quarter Fraction of the 2^k

Table 8-9 Construction of the 2^{6-2}_{IV} Design with the Generators $I = ABCE$ and $I = BCDF$

	Basic Design					
Run	A	B	C	D	$E = ABC$	$F = BCD$
1	−	−	−	−	−	−
2	+	−	−	−	+	−
3	−	+	−	−	+	+
4	+	+	−	−	−	+
5	−	−	+	−	+	+
6	+	−	+	−	−	+
7	−	+	+	−	−	−
8	+	+	+	−	+	−
9	−	−	−	+	−	+
10	+	−	−	+	+	+
11	−	+	−	+	+	−
12	+	+	−	+	−	−
13	−	−	+	+	+	−
14	+	−	+	+	−	−
15	−	+	+	+	−	+
16	+	+	+	+	+	+

The One-Quarter Fraction of the 2^{6-2}

Complete defining relation: $I = ABCE = BCDF = ADEF$

Table 8-8 Alias Structure for the 2_{IV}^{6-2} Design with I = ABCE = BCDF = ADEF

$A = BCE = DEF = ABCDF$	$AB = CE = ACDF = BDEF$
$B = ACE = CDF = ABDEF$	$AC = BE = ABDF = CDEF$
$C = ABE = BDF = ACDEF$	$AD = EF = BCDE = ABCF$
$D = BCF = AEF = ABCDE$	$AE = BC = DF = ABCDEF$
$E = ABC = ADF = BCDEF$	$AF = DE = BCEF = ABCD$
$F = BCD = ADE = ABCEF$	$BD = CF = ACDE = ABEF$
	$BF = CD = ACEF = ABDE$
$ABD = CDE = ACF = BEF$	
$ACD = BDE = ABF = CEF$	

The One-Quarter Fraction of the 2^{6-2}

- Uses of the **alternate** fractions

$$E = \pm ABC, \quad F = \pm BCD$$

- **Projection** of the design into subsets of the original six variables
- Any subset of the original six variables that is not a word in the complete defining relation will result in a full factorial design
  - Consider *ABCD* (full factorial)
  - Consider *ABCE* (replicated half fraction)
  - Consider *ABCF* (full factorial)

A One-Quarter Fraction of the 2^{6-2}: Example 8-4, Page 298

- Injection molding process with six factors
- Design matrix, page 299
- Calculation of effects, normal probability plot of effects
- Two factors (*A*, *B*) and the *AB* interaction are important
- Residual analysis indicates there are some **dispersion effects** (see page 300)

The General 2^{k-p} Fractional Factorial Design

- Section 8-4, page 303
- 2^{k-1} = one-half fraction, 2^{k-2} = one-quarter fraction, 2^{k-3} = one-eighth fraction, …, $2^{k-p} = 1/\ 2^p$ fraction
- Add p columns to the basic design; select p independent generators
- Important to select generators so as to **maximize resolution**, see Table 8-14 page 304
- **Projection** (page 306) – a design of resolution R contains full factorials in any $R - 1$ of the factors
- **Blocking** (page 307)

The General 2^{k-p} Design: Resolution may not be Sufficient

- Minimum abberation designs

Table 8-13 Three Choices of Generators for the 2^{7-2}_{IV} Design

Design A Generators: $F = ABC$, $G = BCD$ $I = ABCF = BCDG = ADFG$	Design B Generators: $F = ABC, G = ADE$ $I = ABCF = ADEG = BCDEFG$	Design C Generators: $F = ABCD$, $G = ABDE$ $I = ABCDF = ABDEG = CEFG$
Aliases (Two-Factor Interactions)	Aliases (Two-Factor Interactions)	Aliases (Two-Factor Interactions)
$AB = CF$	$AB = CF$	$CE = FG$
$AC = BF$	$AC = BF$	$CF = EG$
$AD = FG$	$AD = EG$	$CG = EF$
$AG = DF$	$AE = DG$	
$BD = CG$	$AF = BC$	
$BG = CD$	$AG = DE$	
$AF = BC = DG$		

Resolution III Designs: Section 8-5, page 312

- Designs with main effects aliased with two-factor interactions
- Used for **screening** (5 – 7 variables in 8 runs, 9 - 15 variables in 16 runs, for example)
- A **saturated** design has $k = N - 1$ variables
- See Table 8-19, page 313 for a 2_{III}^{7-4}

Resolution III Designs

Table 8-19 The 2_{III}^{7-4} Design with the Generators $I = ABD$, $I = ACE$, $I = BCF$, and $I = ABCG$

	Basic Design							
Run	A	B	C	$D = AB$	$E = AC$	$F = BC$	$G = ABC$	
1	−	−	−	+	+	+	−	*def*
2	+	−	−	−	−	+	+	*afg*
3	−	+	−	−	+	−	+	*beg*
4	+	+	−	+	−	−	−	*abd*
5	−	−	+	+	−	−	+	*cdg*
6	+	−	+	−	+	−	−	*ace*
7	−	+	+	−	−	+	−	*bcf*
8	+	+	+	+	+	+	+	*abcdefg*

$$[A] \rightarrow A + BD + CE + FG$$
$$[B] \rightarrow B + AD + CF + EG$$
$$[C] \rightarrow C + AE + BF + DG$$
$$[D] \rightarrow D + AB + CG + EF$$
$$[E] \rightarrow E + AC + BG + DF$$
$$[F] \rightarrow F + BC + AG + DE$$
$$[G] \rightarrow G + CD + BE + AF$$

Resolution III Designs

- Sequential assembly of fractions to separate aliased effects (page 315)
- Switching the signs in **one** column provides estimates of that factor and all of its two-factor interactions
- Switching the signs in **all** columns dealiases all main effects from their two-factor interaction alias chains – called a **full fold-over**
- Defining relation for a fold-over (page 318)
- Be **careful** – these rules only work for Resolution III designs
- There are other rules for Resolution IV designs, and other methods for adding runs to fractions to dealias effects of interest
- Example 8-7, eye focus time, page 315

Table 8-21 A 2_{III}^{7-4} Design for the Eye Focus Time Experiment

Run	Basic Design A	B	C	D = AB	E = AC	F = BC	G = ABC		Time
1	−	−	−	+	+	+	−	*def*	85.5
2	+	−	−	−	−	+	+	*afg*	75.1
3	−	+	−	−	+	−	+	*beg*	93.2
4	+	+	−	+	−	−	−	*abd*	145.4
5	−	−	+	+	−	−	+	*cdg*	83.7
6	+	−	+	−	+	−	−	*ace*	77.6
7	−	+	+	−	−	+	−	*bcf*	95.0
8	+	+	+	+	+	+	+	*abcdefg*	141.8

$$[A] = 20.63 \rightarrow A + BD + CE + FG$$
$$[B] = 38.38 \rightarrow B + AD + CF + EG$$
$$[C] = -0.28 \rightarrow C + AE + BF + DG$$
$$[D] = 28.88 \rightarrow D + AB + CG + EF$$
$$[E] = -0.28 \rightarrow E + AC + BG + DF$$
$$[F] = -0.63 \rightarrow F + BC + AG + DE$$
$$[G] = -2.43 \rightarrow G + CD + BE + AF$$

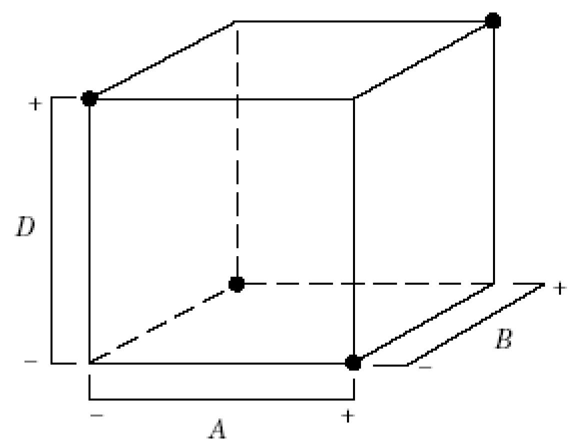


Figure 8-23 The 2_{III}^{7-4} design projected into two replicates of a 2_{III}^{3-1} design in A, B, and D.

Table 8-22 A Fold-Over 2_{III}^{7-4} Design for the Eye Focus Experiment

Run	Basic Design A	B	C	$D = -AB$	$E = -AC$	$F = -BC$	$G = ABC$		Time
1	+	+	+	−	−	−	+	*abcg*	91.3
2	−	+	+	+	+	−	−	*bcde*	136.7
3	+	−	+	+	−	+	−	*acdf*	82.4
4	−	−	+	−	+	+	+	*cefg*	73.4
5	+	+	−	−	+	+	−	*abef*	94.1
6	−	+	−	+	−	+	+	*bdfg*	143.8
7	+	−	−	+	+	−	+	*adeg*	87.3
8	−	−	−	−	−	−	−	(1)	71.9

$$[A]' = -17.68 \rightarrow A - BD - CE - FG$$
$$[B]' = 37.73 \rightarrow B - AD - CF - EG$$
$$[C]' = -3.33 \rightarrow C - AE - BF - DG$$
$$[D]' = 29.88 \rightarrow D - AB - CG - EF$$
$$[E]' = 0.53 \rightarrow E - AC - BG - DF$$
$$[F]' = 1.63 \rightarrow F - BC - AG - DE$$
$$[G]' = 2.68 \rightarrow G - CD - BE - AF$$

i	From $\frac{1}{2}([i] + [i]')$	From $\frac{1}{2}([i] - [i]')$
A	$A = 1.48$	$\mathbf{BD} + CE + FG = 19.15$
B	$\mathbf{B} = 38.05$	$AD + CF + EG = 0.33$
C	$C = -1.80$	$AE + BF + DG = 1.53$
D	$\mathbf{D} = 29.38$	$AB + CG + EF = -0.50$
E	$E = 0.13$	$AC + BG + DF = -0.40$
F	$F = 0.50$	$BC + AG + DE = -1.53$
G	$G = 0.13$	$CD + BE + AF = -2.55$

Remember that the full fold-over technique illustrated in this example (running a “mirror image” design with all signs reversed) only works in a Resolution II design.

Defining relation for a fold-over design – see page 318.

Blocking can be an important consideration in a fold-over design – see page 318.

Plackett-Burman Designs

- These are a different class of resolution III design
- The number of runs, N, need only be a multiple of four
- $N = 4, 8, 12, 16, 20, 24, 28, 32, 36, 40, \ldots$
- The designs where $N = 12, 20, 24$, etc. are called **nongeometric** PB designs
- See text, page 319 for comments on construction of Plackett-Burman designs

Plackett-Burman Designs

Table 8-25 Plackett–Burman Design for Example 8-8

Run	*A*	*B*	*C*	*D*	*E*	*F*	*G*	*H*	*J*	*K*	*L*	Response
1	+	+	−	+	+	+	−	−	−	+	−	231
2	−	+	+	−	+	+	+	−	−	−	+	207
3	+	−	+	+	−	+	+	+	−	−	−	230
4	−	+	−	+	+	−	+	+	+	−	−	217
5	−	−	+	−	+	+	−	+	+	+	−	175
6	−	−	−	+	−	+	+	−	+	+	+	176
7	+	−	−	−	+	−	+	+	−	+	+	183
8	+	+	−	−	−	+	−	+	+	−	+	185
9	+	+	+	−	−	−	+	−	+	+	−	181
10	−	+	+	+	−	−	−	+	−	+	+	220
11	+	−	+	+	+	−	−	−	+	−	+	229
12	−	−	−	−	−	−	−	−	−	−	−	168

See the analysis of this data, page 321

Many effects are large.

Plackett-Burman Designs

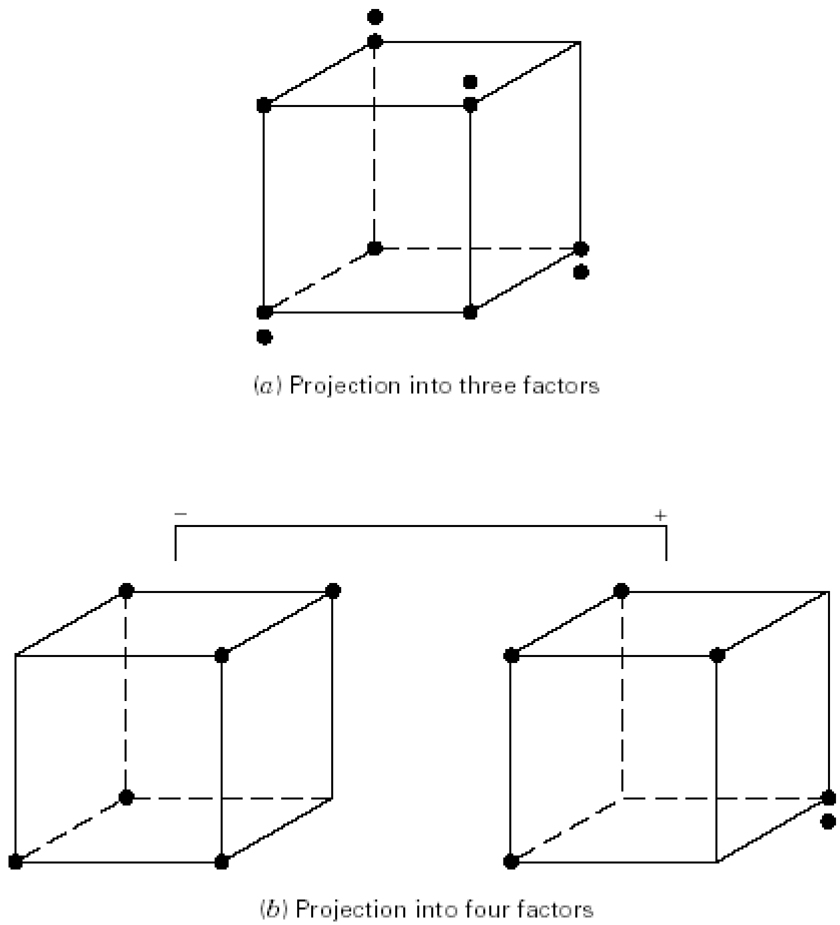


Figure 8-24 Projection of the 12-run Plackett–Burman design into three- and four-factor designs.

Projection of the 12-run design into 3 and 4 factors

All PB designs have **projectivity** 3 (contrast with other resolution III fractions)

Plackett-Burman Designs

- The alias structure is **complex** in the PB designs
- For example, with $N = 12$ and $k = 11$, every main effect is aliased with every 2FI not involving itself
- Every 2FI alias chain has **45** terms
- **Partial** aliasing can greatly complicate interpretation
- Interactions can be particularly disruptive
- Use very, very carefully (maybe never)

Resolution IV and V Designs (Page 322)

Table 8-28 Useful Factorial and Fractional Factorial Designs from the 2^{k-p} System. The Numbers in the Cells Are the Numbers of Factors in the Experiment

	Number of Runs			
Design Type	4	8	16	32
Full factorial	2	3	4	5
Half-fraction	3	4	5	6
Resolution IV fraction	—	4	6–8	7–16
Resolution III fraction	3	5–7	9–15	17–31

A resolution IV design must have at least $2k$ runs.

"optimal" designs may occasionally prove useful.

Sequential Experimentation with Resolution IV Designs – Page 325

Montgomery and Runger (1996) observe that an experimenter may have several objectives in folding over a resolution IV design, such as

1. breaking as many two-factor interaction alias chains as possible,
2. breaking the two-factor interactions on a specific alias chain, or
3. breaking the two-factor interaction aliases involving a specific factor.

We can't use the full fold-over procedure given previously for Resolution III designs – it will result in replicating the runs in the original design.

Switching the signs in a single column allows all of the two-factor interactions involving that column to be separated.

The spin coater experiment – page 326

Table 8-31 The Initial 2_{IV}^{6-2} Design for the Spin Coater Experiment

A	*B*	*C*	*D*	*E*	*F*	
Speed RPM	Acceleration	Vol cc	Time Sec	Resist Viscosity	Exhaust rate	Thickness Mil
−	−	−	−	−	−	4524
+	−	−	−	+	−	4657
−	+	−	−	+	+	4293
+	+	−	−	−	+	4516
−	−	+	−	+	+	4508
+	−	+	−	−	+	4432
−	+	+	−	−	−	4197
+	+	+	−	+	−	4515
−	−	−	+	−	+	4521
+	−	−	+	+	+	4610
−	+	−	+	+	−	4295
+	+	−	+	−	−	4560
−	−	+	+	+	−	4487
+	−	+	+	−	−	4485
−	+	+	+	−	+	4195
+	+	+	+	+	+	4510

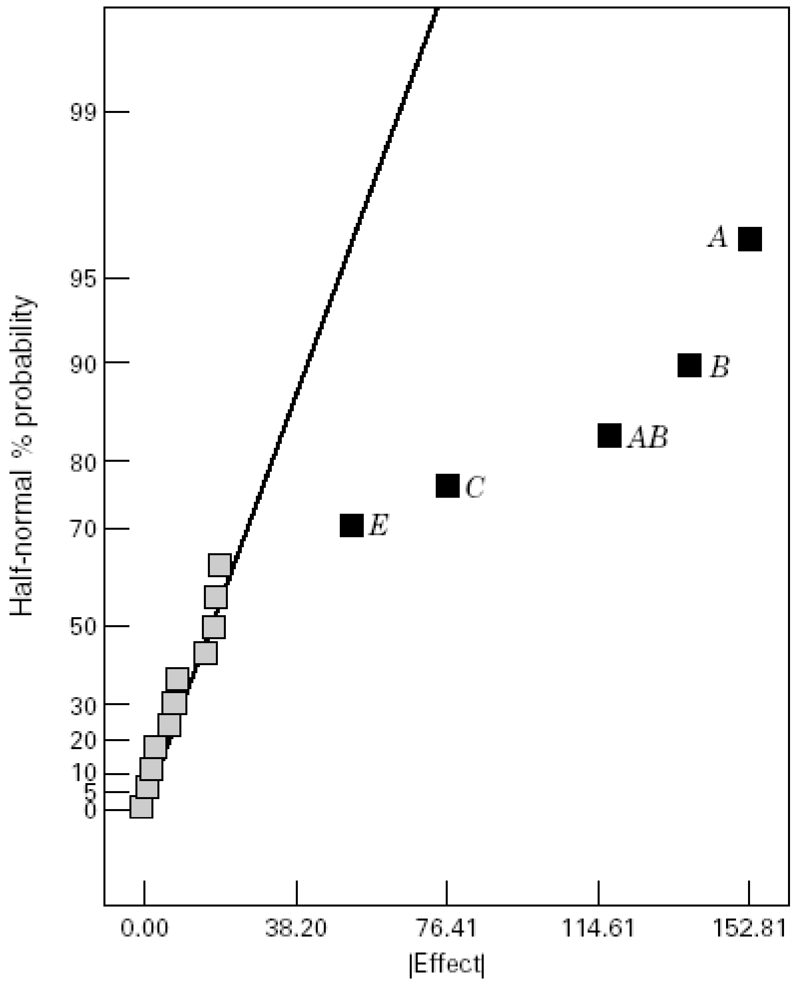


Figure 8-25 Half-normal plot of effects for the initial spin coater experiment in Table 8-31.

[AB] = AB + CE

We need to dealias these interactions

The fold-over design switches the signs in column A

The aliases from the complete design following the fold-over (32 runs) are as follows:

$$
\begin{array}{ll}
[A] = A & [AE] = AE \\
[B] = B & [AF] = AF \\
[C] = C & [BC] = BC + DF \\
[D] = D & [BD] = BD + CF \\
[E] = E & [BE] = BE \\
[F] = F & [BF] = BF + CD \\
[AB] = AB & [CE] = CE \\
[AC] = AC & [DE] = DE \\
[AD] = AD & [EF] = EF
\end{array}
$$

Finding the aliases is somewhat beyond the scope of this course (Chapter 10 provided details) but it can be determined using Design-Expert.

Table 8-32 The Completed Fold Over for the Spin Coater Experiment

		A	*B*	*C*	*D*	*E*	*F*
Std Order	Block	Speed (RPM)	Acceleration	Vol (cc)	Time (Sec)	Resist Viscosity	Exhaus rate
1	1	−	−	−	−	−	−
2	1	+	−	−	−	+	−
3	1	−	+	−	−	+	+
4	1	+	+	−	−	−	+
5	1	−	−	+	−	+	+
6	1	+	−	+	−	−	+
7	1	−	+	+	−	−	−
8	1	+	+	+	−	+	−
9	1	−	−	−	+	−	+
10	1	+	−	−	+	+	+
11	1	−	+	−	+	+	−
12	1	+	+	−	+	−	−
13	1	−	−	+	+	+	−
14	1	+	−	+	+	−	−
15	1	−	+	+	+	−	+
16	1	+	+	+	+	+	+
17	2	+	−	−	−	−	−
18	2	−	−	−	−	+	−
19	2	+	+	−	−	+	+
20	2	−	+	−	−	−	+
21	2	+	−	+	−	+	+
22	2	−	−	+	−	−	+
23	2	+	+	+	−	−	−
24	2	−	+	+	−	+	−
25	2	+	−	−	+	−	+
26	2	−	−	−	+	+	+
27	2	+	+	−	+	+	−
28	2	−	+	−	+	−	−
29	2	+	−	+	+	+	−
30	2	−	−	+	+	−	−
31	2	+	+	+	+	−	+
32	2	−	+	+	+	+	+

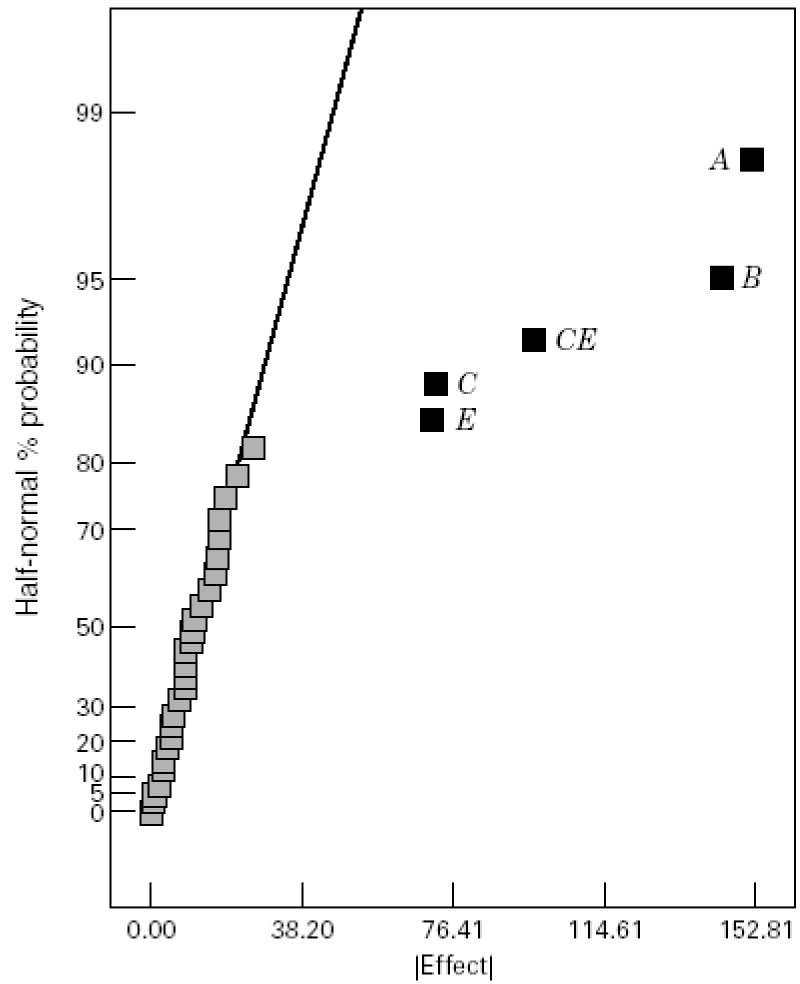


Figure 8-26 Half-normal plot of effects for the spin coater experiment in Table 8-32.

A full fold-over of a Resolution IV design is usually not necessary, and it's potentially very inefficient.

In the spin coater example, there were seven degrees of freedom available to estimate two-factor interaction alias chains.

After adding the fold-over (16 more runs), there are only 12 degrees of freedom available for estimating two-factor interactions (16 new runs yields only five more degrees of freedom).

A **partial fold-over** (semifold) may be a better choice of follow-up design. To construct a partial fold-over:

1. Construct a single-factor fold over from the original design in the usual way by changing the signs on a factor that is involved in a two-factor interaction of interest.
2. Select only half of the fold-over runs by choosing those runs where the chosen factor is either at its high or low level. Selecting the level that you believe will generate the most desirable response is usually a good idea.

Table 8-33 The Partial Fold Over for the Spin Coater Experiment

Std Order	Block	*A* Speed (RPM)	*B* Acceleration	*C* Vol (cc)	*D* Time (Sec)	*E* Resist Viscosity	*F* Exhaust rate	Thickness (Mil)
1	1	−	−	−	−	−	−	4524
2	1	+	−	−	−	+	−	4657
3	1	−	+	−	−	+	+	4293
4	1	+	+	−	−	−	+	4516
5	1	−	−	+	−	+	+	4508
6	1	+	−	+	−	−	+	4432
7	1	−	+	+	−	−	−	4197
8	1	+	+	+	−	+	−	4515
9	1	−	−	−	+	−	+	4521
10	1	+	−	−	+	+	+	4610
11	1	−	+	−	+	+	−	4295
12	1	+	+	−	+	−	−	4560
13	1	−	−	+	+	+	−	4487
14	1	+	−	+	+	−	−	4485
15	1	−	+	+	+	−	+	4195
16	1	+	+	+	+	+	+	4510
17	2	−	−	−	−	+	−	4445
18	2	−	+	−	−	−	+	4285
19	2	−	−	+	−	−	+	4325
20	2	−	+	+	−	+	−	4425
21	2	−	−	−	+	+	+	4525
22	2	−	+	−	+	−	−	4310
23	2	−	−	+	+	−	−	4335
24	2	−	+	+	+	+	+	4305

Not an orthogonal design

Correlated parameter estimates

Larger standard errors of regression model coefficients or effects

$[A] = A$
$[B] = B$
$[C] = C$
$[D] = D$
$[E] = E$
$[F] = F$
$[AB] = AB$
$[AC] = AC$
$[AD] = AD$
$[AE] = AE$
$[AF] = AF$
$[BC] = BC + DF$
$[BD] = BD + CF$
$[BE] = BE$
$[BF] = BF + CD$
$[CE] = CE$
$[DE] = DE$
$[EF] = EF$

There are still 12 degrees of freedom available to estimate two-factor interactions

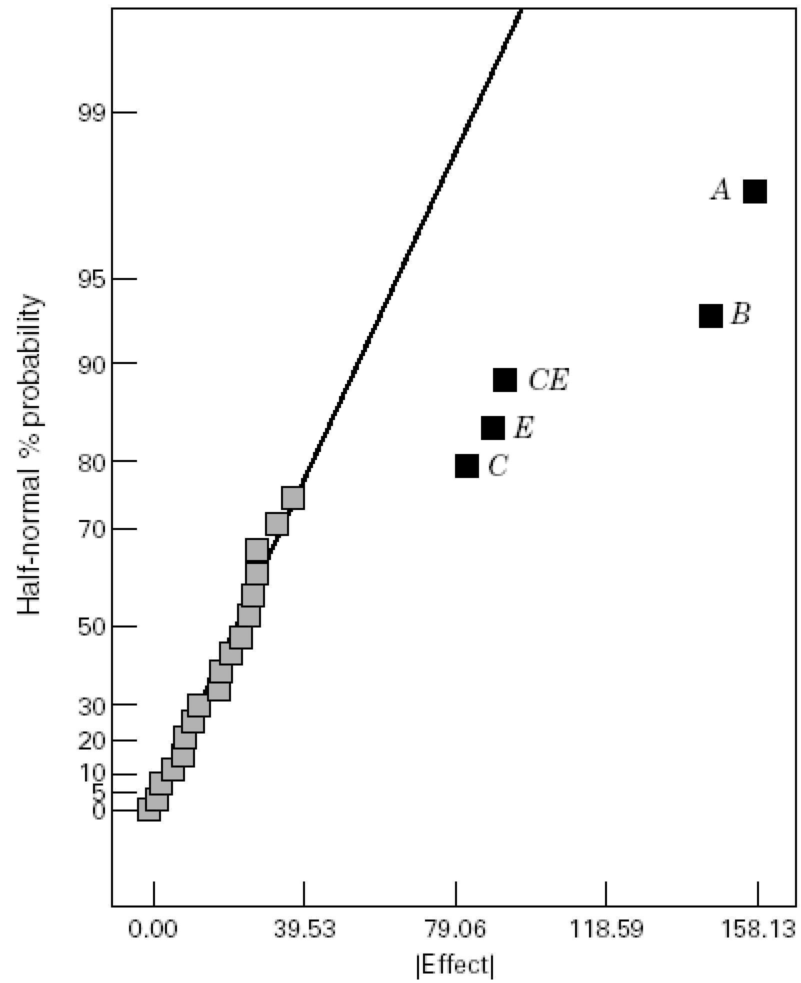


Figure 8-27 Half-normal plot of effects from the partial fold over of the spin coater experiment in Table 8-32.

Resolution V Designs – Page 331

We used a Resolution V design (a 2^{5-2}) in Example 8-2

Generally, these are large designs (at least 32 runs) for six or more factors

Irregular designs can be found using optimal design construction methods

Examples for k = 6 and 8 factors are illustrated in the book